

IB Mathematics HL 12

Vectors, Lines, and Planes Assignment

1. The points $P(-1, 2, -3)$, $Q(-2, 1, 0)$, $R(0, 5, 1)$ and S form a parallelogram, where S is diagonally opposite Q .
 - (a) Find the coordinates of S . [2 marks]
 - (b) The vector product $\overrightarrow{PQ} \times \overrightarrow{PS} = \begin{pmatrix} -13 \\ 7 \\ m \end{pmatrix}$. [2 marks]
Find the value of m .
 - (c) Hence calculate the area of parallelogram PQRS. [2 marks]
 - (d) Find the Cartesian equation of the plane, Π_1 , containing the parallelogram PQRS. [3 marks]
 - (e) Write down the vector equation of the line through the origin $(0, 0, 0)$ that is perpendicular to the plane Π_1 . [1 marks]
 - (f) Hence find the point on the plane that is closest to the origin. [3 marks]
 - (g) A second plane, Π_2 , has equation $x - 2y + z = 3$. [4 marks]
Calculate the angle between the two planes.
2. (a) Consider the vectors $\mathbf{a} = 6\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = -3\mathbf{j} + 4\mathbf{k}$. [11 marks]
 - i. Find the cosine of the angle between vectors \mathbf{a} and \mathbf{b} .
 - ii. Find $\mathbf{a} \times \mathbf{b}$.
 - iii. **Hence** find the Cartesian equation of the plane Π containing the vectors \mathbf{a} and \mathbf{b} and passing through the point $(1, 1, -1)$.
 - iv. The plane Π intersects the x - y plane in the line l . Find the area of the finite triangular region enclosed by l , the x -axis and the y -axis.
- (b) Given two vectors \mathbf{p} and \mathbf{q} , [8 marks]
 - i. show that $\mathbf{p} \cdot \mathbf{p} = |\mathbf{p}|^2$;
 - ii. hence, or otherwise, show that $|\mathbf{p} + \mathbf{q}|^2 = |\mathbf{p}|^2 + 2\mathbf{p} \cdot \mathbf{q} + |\mathbf{q}|^2$;
 - iii. deduce that $|\mathbf{p} + \mathbf{q}| \leq |\mathbf{p}| + |\mathbf{q}|$.