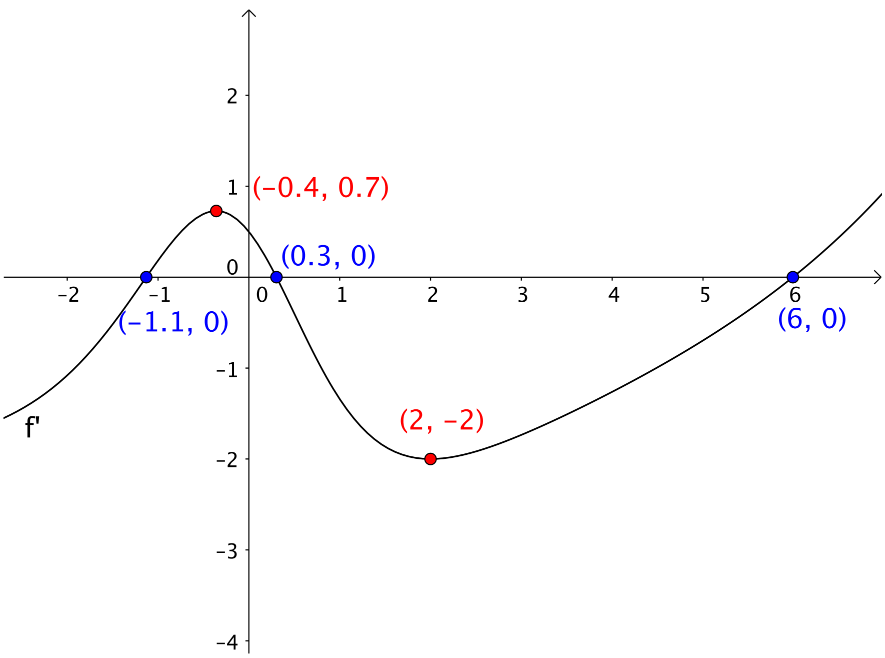
IB Mathematics HL 12
Derivatives and Curve Sketching Assignment

1. Use the definition of the derivative to show that, if $f(x) = -x^2 + 3x + 4$, then $f'(x) = -2x + 3$.

2. Find the equation of the tangent line to the curve $y = f(x)$ at the point on the curve with x-coordinate 1, given that

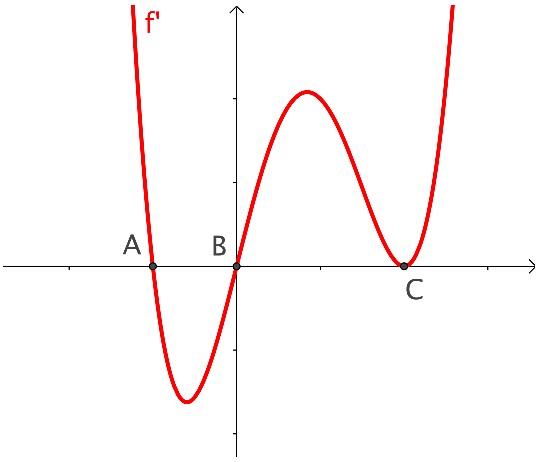
$$f(x) = 2x^3 - \frac{4}{x}.$$

3. Given the graph of f' below, sketch a possible graph of f, clearly indicating the x-coordinates of any points of interest.



ALDAR ACADEMIES
AL BATEEN

4. (★) Let f be a twice-differentiable function with domain $\mathbb{R}$, and assume that f has stationary points at $A(a_1, a_2), B(b_1, b_2), C(c_1, c_2)$ and $D(d_1, d_2)$, with the graph of f' as shown below.



(a) Explain how you know from the graph of f' that $A(a_1, a_2)$ will be a maximum of f. Is $f''(a_1)$ positive or negative?

(b) Explain how you know from the graph of f' that $B(b_1, b_2)$ will be a minimum of f. Is $f''(b_1)$ positive or negative?

(c) Explain how you know from the graph of f' that f has at least three points of inflexion, one of which is $C(c_1, c_2)$. State the value of $f''(c_1)$.

(d) *The Second Derivative Test* provides a way of classifying the stationary points of a twice-differentiable function f based on the value of the second derivative. Complete the statement of the second derivative test below.

Given a twice-differentiable function f with stationary point a,

- if $f''(a) < 0$ then f will have a ________________ at $x = a$.
- if $f''(a) > 0$ then f will have a ________________ at $x = a$.
- if $f''(a) = 0$ then f will have a ________________ at $x = a$.

(e) Given that $f''(d_1) > 0$, is $D(d_1, d_2)$ a maximum, minimum, or point of inflexion of f?

5. (★) Consider the curve given by $y = f(x)$ where

$$f(x) = x^4 - 4x^3 - 2x^2 + 12x$$

(a) Find the derivative of f.

(b) Find an expression for f'', then use the Second Derivative Test to classify the stationary points of f.

(c) Find the coordinates of the (non-stationary) points of inflexion of f.