Unit 11 - Intro to Calculus Review (Ch14&15) *[87 marks]*

1a. *[3 marks]*

Markscheme

intercepts when $f(x) = 0$ ***M1***

(0.827, 0) (4.78, 0) (accept $x = 0.827$, $x = 4.78$) ***A1A1*** ***N3***

[3 marks]

1b. *[1 mark]*

Markscheme

gradient is 1.28 ***A1*** ***N1***

[1 mark]

2a. *[2 marks]*

Markscheme

$f'(x) = 3ax^2 - 12x$ ***A1A1*** ***N2***

Note: Award ***A1*** for each correct term.

[2 marks]

2b. *[4 marks]*

Markscheme

setting their derivative equal to 3 (seen anywhere) ***A1***

e.g. $f'(x) = 3$

attempt to substitute $x = 1$ into $f'(x)$ ***(M1)***

e.g. $3a(1)^2 - 12(1)$

correct substitution into $f'(x)$ ***(A1)***

e.g. $3a - 12$, $3a = 15$

$a = 5$ ***A1*** ***N2***

[4 marks]

3a. *[1 mark]*

Markscheme

$f'(x) = 6e^{6x}$ ***A1*** ***N1***

[1 mark]

3b. *[4 marks]*

Markscheme

(i) evidence of valid approach ***(M1)***

e.g. $f'(0)$, $6e^{6\times 0}$

correct manipulation ***A1***

e.g. $6e^0$, 6×1

$m = 6$ ***AG*** ***N0***

(ii) evidence of finding $f(0)$ ***(M1)***

e.g. $y = e^{6(0)}$

$b = 1$ ***A1*** ***N2***

[4 marks]

3c. *[1 mark]*

Markscheme

$y = 6x + 1$ ***A1*** ***N1***

[1 mark]

4a. *[4 marks]*

Markscheme

evidence of choosing product rule ***(M1)***

eg $uv' + vu'$

correct derivatives (must be seen in the product rule) $\cos x$, $2x$ ***(A1)(A1)***

$f'(x) = x^2 \cos x + 2x \sin x$ ***A1 N4***

[4 marks]

4b. *[3 marks]*

Markscheme

substituting $\frac{\pi}{2}$ into **their** $f'(x)$ ***(M1)***

eg $f'\left(\frac{\pi}{2}\right)$, $\left(\frac{\pi}{2}\right)^2 \cos\left(\frac{\pi}{2}\right) + 2\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)$

correct values for **both** $\sin\frac{\pi}{2}$ and $\cos\frac{\pi}{2}$ seen in $f'(x)$ ***(A1)***

eg $0 + 2\left(\frac{\pi}{2}\right) \times 1$

$f'\left(\frac{\pi}{2}\right) = \pi$ ***A1 N2***

[3 marks]

5. *[4 marks]*

Markscheme

derivative of $2x$ is 2 (must be seen in quotient rule) ***(A1)***

derivative of $x^2 + 5$ is $2x$ (must be seen in quotient rule) ***(A1)***

correct substitution into quotient rule ***A1***

eg $\frac{(x^2+5)(2)-(2x)(2x)}{(x^2+5)^2}$, $\frac{2(x^2+5)-4x^2}{(x^2+5)^2}$

correct working which clearly leads to given answer ***A1***

eg $\frac{2x^2+10-4x^2}{(x^2+5)^2}$, $\frac{2x^2+10-4x^2}{x^4+10x^2+25}$

$f'(x) = \frac{10-2x^2}{(x^2+5)^2}$ ***AG*** ***N0***

[4 marks]

6. *[6 marks]*

Markscheme

recognising need to differentiate (seen anywhere) ***R1***

eg f', $2\mathrm{e}^{2x}$

attempt to find the gradient when $x = 1$ ***(M1)***

eg $f'(1)$

$f'(1) = 2\mathrm{e}^2$ ***(A1)***

attempt to substitute coordinates (in any order) into equation of a straight line ***(M1)***

eg $y - \mathrm{e}^2 = 2\mathrm{e}^2(x-1)$, $\mathrm{e}^2 = 2\mathrm{e}^2(1) + b$

correct working ***(A1)***

eg $y - \mathrm{e}^2 = 2\mathrm{e}^2 x - 2\mathrm{e}^2$, $b = -\mathrm{e}^2$

$y = 2\mathrm{e}^2 x - \mathrm{e}^2$ ***A1*** ***N3***

[6 marks]

7. *[2 marks]*

Markscheme

expressing f as $x^{\frac{4}{3}}$ ***(M1)***

$f'(x) = \frac{4}{3}x^{\frac{1}{3}}$ $\left(= \frac{4}{3}\sqrt[3]{x}\right)$ ***A1*** ***N2***

[2 marks]

8. *[6 marks]*

Markscheme

METHOD 1 (quotient)

derivative of numerator is 6 ***(A1)***

derivative of denominator is $-\sin x$ ***(A1)***

attempt to substitute into quotient rule ***(M1)***

correct substitution ***A1***

e.g. $\frac{(\cos x)(6)-(6x)(-\sin x)}{(\cos x)^2}$

substituting $x=0$ ***(A1)***

e.g. $\frac{(\cos 0)(6)-(6\times 0)(-\sin 0)}{(\cos 0)^2}$

$h'(0)=6$ ***A1*** ***N2***

METHOD 2 (product)

$h(x)=6x\times(\cos x)^{-1}$

derivative of $6x$ is 6 ***(A1)***

derivative of $(\cos x)^{-1}$ is $(-(\cos x)^{-2}(-\sin x))$ ***(A1)***

attempt to substitute into product rule **(M1)**

correct substitution ***A1***

e.g. $(6x)(-(\cos x)^{-2}(-\sin x))+(6)(\cos x)^{-1}$

substituting $x=0$ ***(A1)***

e.g. $(6\times 0)(-(\cos 0)^{-2}(-\sin 0))+(6)(\cos 0)^{-1}$

$h'(0)=6$ ***A1*** ***N2***

[6 marks]

9a. *[4 marks]*

Markscheme

evidence of choosing the product rule ***(M1)***

e.g. $uv'+vu'$

correct derivatives $\cos x$, 2 ***(A1)(A1)***

$g'(x)=2x\cos x+2\sin x$ ***A1*** ***N4***

[4 marks]

9b. *[3 marks]*

Markscheme

attempt to substitute into gradient function ***(M1)***

e.g. $g'(\pi)$

correct substitution ***(A1)***

e.g. $2\pi\cos\pi+2\sin\pi$

gradient $=-2\pi$ ***A1*** ***N2***

[3 marks]

10a. *[5 marks]*

Markscheme

$\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$, $\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$ (seen anywhere) ***(A1)(A1)***

evidence of using the quotient rule ***M1***

correct substitution ***A1***

e.g. $\frac{\sin x(-\sin x)-\cos x(\cos x)}{\sin^2 x}$, $\frac{-\sin^2 x-\cos^2 x}{\sin^2 x}$

$f'(x) = \frac{-(\sin^2 x+\cos^2 x)}{\sin^2 x}$ ***A1***

$f'(x) = \frac{-1}{\sin^2 x}$ ***AG*** ***N0***

[5 marks]

10b. *[3 marks]*

Markscheme

METHOD 1

appropriate approach ***(M1)***

e.g. $f'(x) = -(\sin x)^{-2}$

$f''(x) = 2(\sin^{-3} x)(\cos x)\left(= \frac{2\cos x}{\sin^3 x}\right)$ ***A1A1*** ***N3***

Note: Award ***A1*** for $2\sin^{-3} x$, ***A1*** for $\cos x$.

METHOD 2

derivative of $\sin^2 x = 2\sin x\cos x$ (seen anywhere) ***A1***

evidence of choosing quotient rule ***(M1)***

e.g. $u = -1$, $v = \sin^2 x$, $f'' = \frac{\sin^2 x\times 0-(-1)2\sin x\cos x}{(\sin^2 x)^2}$

$f''(x) = \frac{2\sin x\cos x}{(\sin^2 x)^2}\left(= \frac{2\cos x}{\sin^3 x}\right)$ ***A1*** ***N3***

[3 marks]

11. *[3 marks]*

Markscheme

evidence of choosing the product rule ***(M1)***

e.g. $x\times(-\sin x)+1\times\cos x$

$f'(x) = \cos x - x\sin x$ ***A1A1*** ***N3***

[3 marks]

12a. *[2 marks]*

Markscheme

(a) $f'(x) = -\sin 2x\times 2(= -2\sin 2x)$ ***A1A1*** ***N2***

Note: Award ***A1*** for 2, ***A1*** for $-\sin 2x$.

[2 marks]

12b. *[2 marks]*

Markscheme

$g'(x) = 3 \times \frac{1}{3x-5}\ \left(= \frac{3}{3x-5}\right)$ ***A1A1*** ***N2***

Note: Award ***A1*** for 3, ***A1*** for $\frac{1}{3x-5}$.

[2 marks]

12c. *[2 marks]*

Markscheme

evidence of using product rule ***(M1)***

$h'(x) = (\cos 2x)\left(\frac{3}{3x-5}\right) + \ln(3x-5)(-2\sin 2x)$ ***A1*** ***N2***

[2 marks]

13. *[7 marks]*

Markscheme

substituting $x = 1$, $y = 3$ into $f(x)$ ***(M1)***

$3 = p + q$ ***A1***

finding derivative ***(M1)***

$f'(x) = 2px + q$ ***A1***

correct substitution, $2p + q = 8$ ***A1***

$p = 5$, $q = -2$ ***A1A1*** ***N2N2***

[7 marks]

14. *[6 marks]*

Markscheme

evidence of choosing the product rule ***(M1)***

$f'(x) = \mathrm{e}^x \times (-\sin x) + \cos x \times \mathrm{e}^x\ (= \mathrm{e}^x \cos x - \mathrm{e}^x \sin x)$ ***A1A1***

substituting π ***(M1)***

e.g. $f'(\pi) = \mathrm{e}^\pi \cos \pi - \mathrm{e}^\pi \sin \pi$, $\mathrm{e}^\pi(-1-0)$, $-\mathrm{e}^\pi$

taking negative reciprocal ***(M1)***

e.g. $-\frac{1}{f'(\pi)}$

gradient is $\frac{1}{\mathrm{e}^\pi}$ ***A1*** ***N3***

[6 marks]

15a. *[2 marks]*

Markscheme

attempt to expand ***(M1)***

$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$ ***A1*** ***N2***

[2 marks]

15b. *[4 marks]*

Markscheme

evidence of substituting $x+h$ ***(M1)***

correct substitution ***A1***

e.g. $f'(x)=\lim\limits_{h\to 0}\frac{(x+h)^3-4(x+h)+1-(x^3-4x+1)}{h}$

simplifying ***A1***

e.g. $\frac{(x^3+3x^2h+3xh^2+h^3-4x-4h+1-x^3+4x-1)}{h}$

factoring out h ***A1***

e.g. $\frac{h(3x^2+3xh+h^2-4)}{h}$

$f'(x)=3x^2-4$ ***AG*** ***N0***

[4 marks]

16a. *[2 marks]*

Markscheme

METHOD 1

$f''(x)=3(x-3)^2$ ***A2*** ***N2***

METHOD 2

attempt to expand $(x-3)^3$ ***(M1)***

e.g. $f'(x)=x^3-9x^2+27x-27$

$f''(x)=3x^2-18x+27$ ***A1*** ***N2***

[2 marks]

16b. *[1 mark]*

Markscheme

$f'(3)=0$, $f''(3)=0$ ***A1*** ***N1***

[1 mark]

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