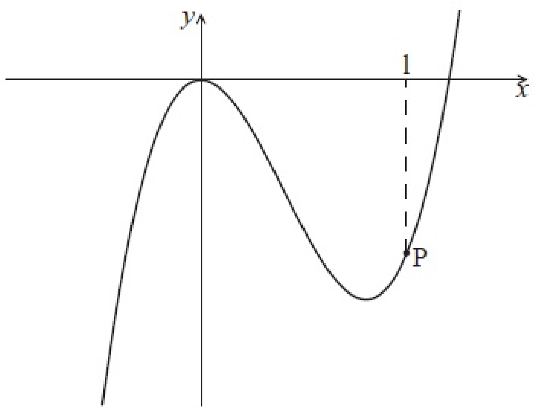
Unit 11 - Intro to Calculus Review (Ch14&15) *[87 marks]*

Let $f(x) = 4x - \mathrm{e}^{x-2} - 3$, for $0 \leq x \leq 5$.

1a. Find the x-intercepts of the graph of f . *[3 marks]*

1b. Write down the gradient of the graph of f at $x = 3$. *[1 mark]*

Part of the graph of $f(x) = ax^3 - 6x^2$ is shown below.



The point P lies on the graph of f . At P, $x = 1$.

2a. Find $f'(x)$. *[2 marks]*

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2b. The graph of f has a gradient of 3 at the point P. Find the value of a . *[4 marks]*

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Let $f(x) = \mathrm{e}^{6x}$.

3a. Write down $f'(x)$. *[1 mark]*

3b. The tangent to the graph of *f* at the point $\mathrm{P}(0, b)$ has gradient m . *[4 marks]*

(i) Show that $m = 6$.

(ii) Find b .

3c. Hence, write down the equation of this tangent. *[1 mark]*

Consider $f(x) = x^2 \sin x$.

4a. Find $f'(x)$. *[4 marks]*

4b. Find the gradient of the curve of f at $x = \frac{\pi}{2}$. *[3 marks]*

Let $f(x) = \frac{2x}{x^2+5}$.

5. Use the quotient rule to show that $f'(x) = \frac{10-2x^2}{(x^2+5)^2}$. *[4 marks]*

6. Let $f(x) = \mathrm{e}^{2x}$. The line L is the tangent to the curve of f at $(1,\ \mathrm{e}^2)$. *[6 marks]*
Find the equation of L in the form $y = ax + b$.

Let $f(x) = \sqrt[3]{x^4} - \frac{1}{2}$.

7. Find $f'(x)$. *[2 marks]*

8. Let $h(x) = \frac{6x}{\cos x}$. Find $h'(0)$. *[6 marks]*

Let $g(x) = 2x \sin x$.

9a. Find $g'(x)$. *[4 marks]*

9b. Find the gradient of the graph of g at $x = \pi$. *[3 marks]*

Let $f(x) = \frac{\cos x}{\sin x}$, for $\sin x \neq 0$.

10a. Use the quotient rule to show that $f'(x) = \frac{-1}{\sin^2 x}$. *[5 marks]*

10b. Find $f''(x)$. *[3 marks]*

Let $f(x) = x\cos x$, for $0 \leq x \leq 6$.

11. Find $f'(x)$. *[3 marks]*

Let $f(x) = \cos 2x$ and $g(x) = \ln(3x - 5)$.

12a. Find $f'(x)$. *[2 marks]*

12b. Find $g'(x)$. *[2 marks]*

12c. Let $h(x) = f(x) \times g(x)$. Find $h'(x)$. *[2 marks]*

13. Consider the curve with equation $f(x) = px^2 + qx$, where p and q are constants. The point $\mathrm{A}(1, 3)$ lies on the curve. The tangent to the curve at A has gradient 8. Find the value of p and of q . *[7 marks]*

14. Let $f(x) = \mathrm{e}^x \cos x$. Find the gradient of the normal to the curve of f at $x = \pi$. *[6 marks]*

Let $f(x) = x^3 - 4x + 1$.

15a. Expand $(x + h)^3$. *[2 marks]*

15b. Use the formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to show that the derivative of $f(x)$ is $3x^2 - 4$. *[4 marks]*

A function f has its first derivative given by $f'(x) = (x-3)^3$.

16a. Find the second derivative. *[2 marks]*

16b. Find $f'(3)$ and $f''(3)$. *[1 mark]*