Exponents and Logarithms Review *[63 marks]*

A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After *n* years the number of taxis, *T*, in the city is given by

$$T = 280 \times 1.12^n.$$

1a. (i) Find the number of taxis in the city at the end of 2005. *[6 marks]*

(ii) Find the year in which the number of taxis is double the number of taxis there were at the end of 2000.

1b. At the end of 2000 there were 25600 people in the city who used taxis. *[6 marks]*

After *n* years the number of people, *P*, in the city who used taxis is given by

$$P = \frac{2560000}{10 + 90\mathrm{e}^{-0.1n}}.$$

(i) Find the value of *P* at the end of 2005, giving your answer to the nearest whole number.

(ii) After seven complete years, will the value of *P* be double its value at the end of 2000? Justify your answer.

1c. Let *R* be the ratio of the number of people using taxis in the city to the number of taxis. The city will reduce the number of taxis if $R < 70$. *[5 marks]*

(i) Find the value of *R* at the end of 2000.

(ii) After how many complete years will the city first reduce the number of taxis?

Let $f(x) = \mathrm{e}^{x+3}$.

2a. (i) Show that $f^{-1}(x) = \ln x - 3$. *[3 marks]*

(ii) Write down the domain of f^{-1} .

2b. Solve the equation $f^{-1}(x) = \ln \frac{1}{x}$. *[4 marks]*

A population of rare birds, P_t, can be modelled by the equation $P_t = P_0e^{kt}$, where P_0 is the initial population, and t is measured in decades. After one decade, it is estimated that $\frac{P_1}{P_0} = 0.9$.

3a. (i) Find the value of k. *[3 marks]*

(ii) Interpret the meaning of the value of k.

3b. Find the least number of **whole** years for which $\frac{P_t}{P_0} < 0.75$. *[5 marks]*

4a. Find the value of $\log_2 40 - \log_2 5$. *[3 marks]*

4b. Find the value of $8^{\log_2 5}$. *[4 marks]*

Jose takes medication. After *t* minutes, the concentration of medication left in his bloodstream is given by $A(t) = 10(0.5)^{0.014t}$, where *A* is in milligrams per litre.

5a. Write down $A(0)$. *[1 mark]*

5b. Find the concentration of medication left in his bloodstream after 50 minutes. *[2 marks]*

5c. At 13:00, when there is no medication in Jose's bloodstream, he takes his first dose of medication. He can take his medication again when the concentration of medication reaches 0.395 milligrams per litre. What time will Jose be able to take his medication again? *[5 marks]*

Let
$f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for
$x > 0$.

6a. Show that $f(x) = \log_3 2x$. *[2 marks]*

6b. Find the value of $f(0.5)$ and of $f(4.5)$. *[3 marks]*

6c. The function *f* can also be written in the form $f(x) = \frac{\ln ax}{\ln b}$. *[6 marks]*

(i) Write down the value of *a* and of *b* .

(ii) Hence on graph paper, **sketch** the graph of *f* , for
$-5 \leq x \leq 5$,
$-5 \leq y \leq 5$, using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote.

6d. Write down the value of $f^{-1}(0)$. *[1 mark]*

6e. The point A lies on the graph of *f* . At A, $x = 4.5$. *[4 marks]*

On your diagram, sketch the graph of f^{-1} , noting clearly the image of point A.