Roots of Complex Numbers

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Objective(s): ▷ to extend the formula to find the n^{th} roots of unity in order to find the n^{th} roots of an arbitrary complex number

Roots of Unity

For $n \in \mathbb{Z}^+$, the n^{th} *roots of unity* are the complex numbers that are solutions to the equation

$$z^n = 1, \text{ or equivalently}$$

$$z^n - 1 = 0.$$

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The n^{th} roots of unity are given by

$$e^{\mathrm{i}\left(\frac{2k\pi}{n}\right)}, \text{ for } k = 0, \ldots, n-1$$

General Roots

1. There are three cube roots of 8.
   a) Find an equation that gives the arguments of all cube roots of 8.
   b) Plot the cube roots of 8 on the Argand diagram.
2. There are three cube roots of i.
   a) Find an equation that gives the arguments of all cube roots of i.
   b) Plot the cube roots of i on the Argand diagram.

General Roots

3. The complex number $e^{i\left(\frac{\pi}{3}\right)}$ has three cube roots. Find each of the roots and express your answers in Euler form. Plot your answers on the Argand diagram.
4. The complex number $8e^{i\left(\frac{\pi}{3}\right)}$ has three cube roots. Find each of the roots and express your answers in Euler form. Plot your answers on the Argand diagram.
5. The complex number $1 + 2i$ has five fifth roots. Find each of the roots and express your answers in Cartesian form, with values accurate to 3 decimal places. Plot your answers on the Argand diagram.

Can you find a general expression for the n^{th} roots of an arbitrary complex number $c \in \mathbb{C}$? (Consider c to be of the form $re^{\mathrm{i}\theta}$.)

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Roots of a Complex Number

Given $c \in \mathbb{C}$, with $c = re^{i\theta}$, and some positive integer n, the n^{th} roots of c are solutions to the equation

$$z^n = c$$

There are n such roots, each with modulus $r^{\frac{1}{n}}$, and with arguments given by

$$\frac{\theta + 2k\pi}{n}, \text{ for } k = 0, \ldots, n-1$$

Questions

Complete

Exercise 14G.1 questions 2, 5, 6ab, 8
Exercise 14G.2 questions 1a, 3

Difficult Challenge Explain how the assumption in question 3b (that ω is the root with least positive argument) can be loosened. What must we assume about ω in order for all of the fifth roots of unity to be represented by $1, \omega, \omega^2, \omega^3$, and ω^4?