Intro to Calculus Review *[158 marks]*

The values of the functions f and g and their derivatives for $x = 1$ and $x = 8$ are shown in the following table.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	4	9	–3
8	4	–3	2	5

Let $h(x) = f(x)g(x)$.

1a. Find $h(1)$. *[2 marks]*

Markscheme

expressing $h(1)$ as a product of $f(1)$ and $g(1)$ ***(A1)***

eg $f(1) \times g(1), 2(9)$

$h(1) = 18$ ***A1 N2***

[2 marks]

1b. Find $h'(8)$. *[3 marks]*

Markscheme

attempt to use product rule (do **not** accept $h' = f' \times g'$) ***(M1)***

eg $h' = fg' + gf', h'(8) = f'(8)g(8) + g'(8)f(8)$

correct substitution of values into product rule ***(A1)***

eg $h'(8) = 4(5) + 2(-3), \ -6 + 20$

$h'(8) = 14$ ***A1 N2***

[3 marks]

2. Let $f(x) = (x^2 + 3)^7$. Find the term in x^5 in the expansion of the derivative, $f'(x)$. *[7 marks]*

Markscheme

METHOD 1

derivative of $f(x)$ ***A2***

$7(x^2+3)^6(x2)$

recognizing need to find x^4 term in $(x^2+3)^6$ (seen anywhere) ***R1***

eg $14x\ (\text{term in } x^4)$

valid approach to find the terms in $(x^2+3)^6$ ***(M1)***

eg $\binom{6}{r}(x^2)^{6-r}(3)^r,\ (x^2)^6(3)^0+(x^2)^5(3)^1+\ldots$, Pascal's triangle to 6th row

identifying correct term (may be indicated in expansion) ***(A1)***

eg $5\text{th term},\ r=2,\ \binom{6}{4},\ (x^2)^2(3)^4$

correct working (may be seen in expansion) ***(A1)***

eg $\binom{6}{4}(x^2)^2(3)^4,\ 15\times3^4,\ 14x\times15\times81(x^2)^2$

$17010x^5$ ***A1*** ***N3***

METHOD 2

recognition of need to find x^6 in $(x^2+3)^7$ (seen anywhere) ***R1***

valid approach to find the terms in $(x^2+3)^7$ ***(M1)***

eg $\binom{7}{r}(x^2)^{7-r}(3)^r,\ (x^2)^7(3)^0+(x^2)^6(3)^1+\ldots$, Pascal's triangle to 7th row

identifying correct term (may be indicated in expansion) ***(A1)***

eg 6th term, $r=3,\ \binom{7}{3},\ (\mathrm{x}^2)^3(3)^4$

correct working (may be seen in expansion) ***(A1)***

eg $\binom{7}{4}(\mathrm{x}^2)^3(3)^4,\ 35\times3^4$

correct term ***(A1)***

$2835x^6$

differentiating their term in x^6 ***(M1)***

eg $(2835x^6)'$, $(6)(2835\mathrm{x}^5)$

$17010x^5$ ***A1*** ***N3***

[7 marks]

3. Let $f(x) = \frac{\ln(4x)}{x}$ for $0 < x \leq 5$. *[7 marks]*

Points $\mathrm{P}(0.25, 0)$ and Q are on the curve of f. The tangent to the curve of f at P is perpendicular to the tangent at Q. Find the coordinates of Q.

Markscheme

recognizing that the gradient of tangent is the derivative ***(M1)***

eg f'

finding the gradient of f at P ***(A1)***

eg $f'(0.25) = 16$

evidence of taking negative reciprocal of **their** gradient at P ***(M1)***

eg $\frac{-1}{m}$, $-\frac{1}{f'(0.25)}$

equating derivatives ***M1***

eg $f'(x) = \frac{-1}{16}, f' = -\frac{1}{m}, \frac{x\left(\frac{1}{x}\right)-\ln(4x)}{x^2} = 16$

finding the x-coordinate of Q, $x = 0.700750$

$x = 0.701$ ***A1 N3***

attempt to substitute **their** x into f to find the y-coordinate of Q ***(M1)***

eg $f(0.7)$

$y = 1.47083$

$y = 1.47$ ***A1 N2***

[7 marks]

Let $f(x) = \frac{(\ln x)^2}{2}$, for $x > 0$.

4a. Show that $f'(x) = \frac{\ln x}{x}$. *[2 marks]*

Markscheme

METHOD 1

correct use of chain rule ***A1A1***

eg $\frac{2\ln x}{2} \times \frac{1}{x}, \frac{2\ln x}{2x}$

Note: Award ***A1*** for $\frac{2\ln x}{2x}$, ***A1*** for $\times\frac{1}{x}$.

$f'(x) = \frac{\ln x}{x}$ ***AG N0***

[2 marks]

METHOD 2

correct substitution into quotient rule, with derivatives seen ***A1***

eg $\frac{2\times 2\ln x \times \frac{1}{x} - 0\times(\ln x)^2}{4}$

correct working ***A1***

eg $\frac{4\ln x \times \frac{1}{x}}{4}$

$f'(x) = \frac{\ln x}{x}$ ***AG N0***

[2 marks]

4b. There is a minimum on the graph of f. Find the x-coordinate of this minimum. *[3 marks]*

Markscheme

setting derivative $= 0$ ***(M1)***

eg $f'(x) = 0, \frac{\ln x}{x} = 0$

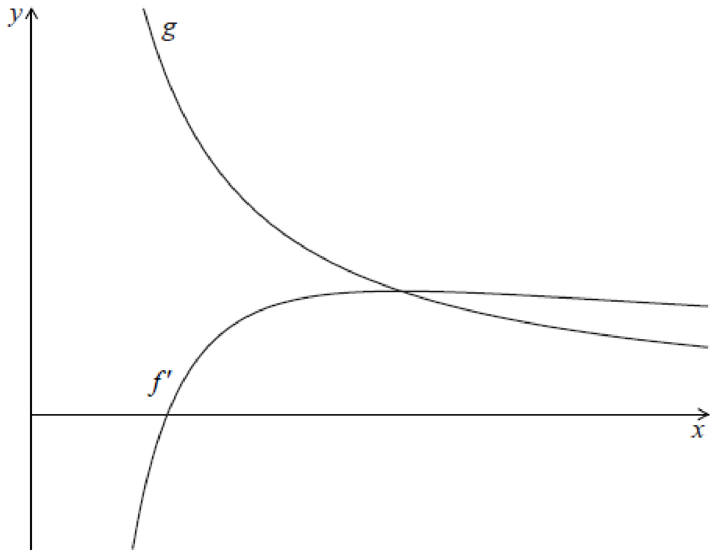
correct working ***(A1)***

eg $\ln x = 0, x = \mathrm{e}^0$

$x = 1$ ***A1 N2***

[3 marks]

Let $g(x) = \frac{1}{x}$. The following diagram shows parts of the graphs of f' and g.



The graph of f' has an x-intercept at $x = p$.

4c. Write down the value of p. *[2 marks]*

Markscheme

intercept when $f'(x) = 0$ ***(M1)***

$p = 1$ ***A1 N2***

[2 marks]

4d. The graph of g intersects the graph of f' when $x = q$. *[3 marks]*

Find the value of q.

Markscheme

equating functions ***(M1)***

eg $f' = g, \frac{\ln x}{x} = \frac{1}{x}$

correct working ***(A1)***

eg $\ln x = 1$

$q = \mathrm{e}\ (\text{accept} x = \mathrm{e})$ ***A1 N2***

[3 marks]

5. Let $f(x) = \mathrm{e}^{2x}$. The line L is the tangent to the curve of f at $(1, \mathrm{e}^2)$. *[6 marks]*

Find the equation of L in the form $y = ax + b$.

Markscheme

recognising need to differentiate (seen anywhere) ***R1***

eg $f', 2\mathrm{e}^{2x}$

attempt to find the gradient when $x = 1$ ***(M1)***

eg $f'(1)$

$f'(1) = 2\mathrm{e}^2$ ***(A1)***

attempt to substitute coordinates (in any order) into equation of a straight line ***(M1)***

eg $y - \mathrm{e}^2 = 2\mathrm{e}^2(x-1), \mathrm{e}^2 = 2\mathrm{e}^2(1) + b$

correct working ***(A1)***

eg $y - \mathrm{e}^2 = 2\mathrm{e}^2 x - 2\mathrm{e}^2, b = -\mathrm{e}^2$

$y = 2\mathrm{e}^2 x - \mathrm{e}^2$ ***A1 N3***

[6 marks]

Let $f(x) = \sqrt[3]{x^4} - \frac{1}{2}$.

6. Find $f'(x)$. *[2 marks]*

Markscheme

expressing f as $x^{\frac{4}{3}}$ ***(M1)***

$f'(x) = \frac{4}{3}x^{\frac{1}{3}} \left(= \frac{4}{3}\sqrt[3]{x}\right)$ ***A1 N2***

[2 marks]

Consider $f(x) = x^2 \sin x$.

7a. Find $f'(x)$. *[4 marks]*

Markscheme

evidence of choosing product rule ***(M1)***

eg $uv' + vu'$

correct derivatives (must be seen in the product rule) $\cos x$, $2x$ ***(A1)(A1)***

$f'(x) = x^2 \cos x + 2x \sin x$ ***A1 N4***

[4 marks]

7b. Find the gradient of the curve of f at $x = \frac{\pi}{2}$. *[3 marks]*

Markscheme

substituting $\frac{\pi}{2}$ into **their** $f'(x)$ ***(M1)***

eg $f'\left(\frac{\pi}{2}\right)$, $\left(\frac{\pi}{2}\right)^2 \cos\left(\frac{\pi}{2}\right) + 2\left(\frac{\pi}{2}\right) \sin\left(\frac{\pi}{2}\right)$

correct values for **both** $\sin \frac{\pi}{2}$ and $\cos \frac{\pi}{2}$ seen in $f'(x)$ ***(A1)***

eg $0 + 2\left(\frac{\pi}{2}\right) \times 1$

$f'\left(\frac{\pi}{2}\right) = \pi$ ***A1 N2***

[3 marks]

Let $f(x) = \sin x + \frac{1}{2}x^2 - 2x$, for $0 \leq x \leq \pi$.

8a. Find $f'(x)$. *[3 marks]*

Markscheme

$f'(x) = \cos x + x - 2$ ***A1A1A1 N3***

Note: Award ***A1*** for each term.

[3 marks]

Let g be a quadratic function such that $g(0) = 5$. The line $x = 2$ is the axis of symmetry of the graph of g .

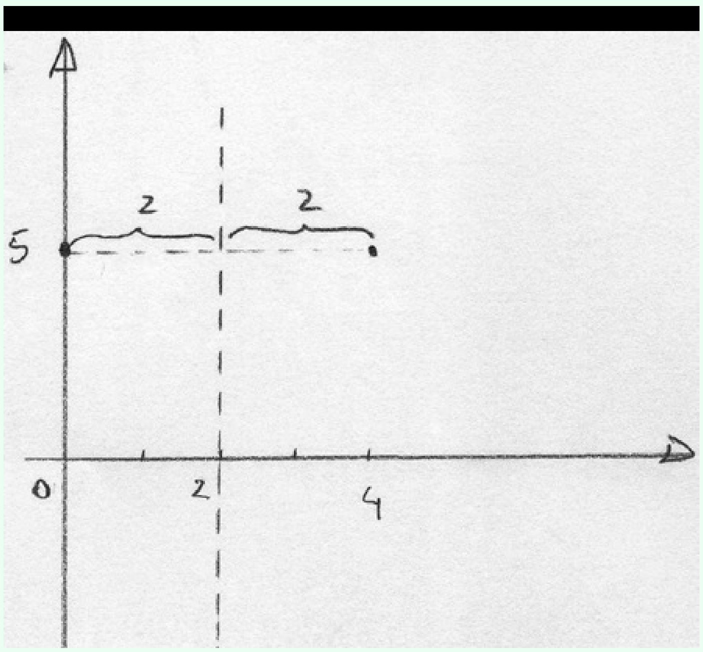
8b. Find $g(4)$. *[3 marks]*

Markscheme

recognizing $g(0) = 5$ gives the point $(0,\ 5)$ ***(R1)***

recognize symmetry ***(M1)***

eg vertex, sketch



$g(4) = 5$ ***A1 N3***

[3 marks]

The function g can be expressed in the form $g(x) = a(x - h)^2 + 3$.

8c. (i) Write down the value of h . *[4 marks]*

(ii) Find the value of a .

Markscheme

(i) $h = 2$ ***A1 N1***

(ii) substituting into $g(x) = a(x-2)^2 + 3$ (not the vertex) ***(M1)***

eg $5 = a(0-2)^2 + 3$, $5 = a(4-2)^2 + 3$

working towards solution ***(A1)***

eg $5 = 4a + 3$, $4a = 2$

$a = \frac{1}{2}$ ***A1 N2***

[4 marks]

8d. Find the value of x for which the tangent to the graph of f is parallel to the tangent to the graph of g . *[6 marks]*

Markscheme

$g(x) = \frac{1}{2}(x-2)^2 + 3 = \frac{1}{2}x^2 - 2x + 5$

correct derivative of g ***A1A1***

eg $2 \times \frac{1}{2}(x-2)$, $x - 2$

evidence of equating both derivatives ***(M1)***

eg $f' = g'$

correct equation ***(A1)***

eg $\cos x + x - 2 = x - 2$

working towards a solution ***(A1)***

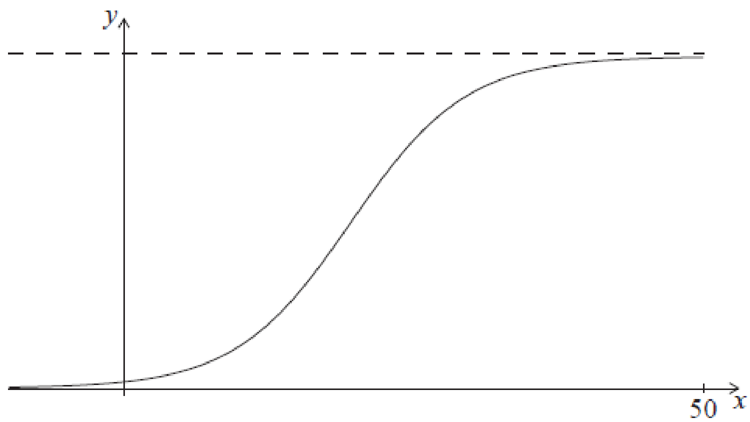
eg $\cos x = 0$, combining like terms

$x = \frac{\pi}{2}$ ***A1 N0***

Note: Do not award final ***A1*** if additional values are given.

[6 marks]

Let $f(x) = \frac{100}{(1+50e^{-0.2x})}$. Part of the graph of f is shown below.



9a. Write down $f(0)$. *[1 mark]*

Markscheme

$f(0) = \frac{100}{51}$ (exact), 1.96 ***A1 N1***

[1 mark]

9b. Solve $f(x) = 95$. *[2 marks]*

Markscheme

setting up equation ***(M1)***

eg $95 = \frac{100}{1+50e^{-0.2x}}$, sketch of graph with horizontal line at $y = 95$

$x = 34.3$ ***A1 N2***

[2 marks]

9c. Find the range of f . *[3 marks]*

Markscheme

upper bound of y is 100 ***(A1)***

lower bound of y is 0 ***(A1)***

range is $0 < y < 100$ ***A1 N3***

[3 marks]

9d. Show that $f'(x) = \frac{1000e^{-0.2x}}{(1+50e^{-0.2x})^2}$. *[5 marks]*

Markscheme

METHOD 1

setting function ready to apply the chain rule ***(M1)***

eg $100(1+50\mathrm{e}^{-0.2x})^{-1}$

evidence of correct differentiation (must be substituted into chain rule) ***(A1)(A1)***

eg $u' = -100(1+50\mathrm{e}^{-0.2x})^{-2}$, $v' = (50\mathrm{e}^{-0.2x})(-0.2)$

correct chain rule derivative ***A1***

eg $f'(x) = -100(1+50\mathrm{e}^{-0.2x})^{-2}(50\mathrm{e}^{-0.2x})(-0.2)$

correct working clearly leading to the required answer ***A1***

eg $f'(x) = 1000\mathrm{e}^{-0.2x}(1+50\mathrm{e}^{-0.2x})^{-2}$

$f'(x) = \frac{1000\mathrm{e}^{-0.2x}}{(1+50\mathrm{e}^{-0.2x})^2}$ ***AG N0***

METHOD 2

attempt to apply the quotient rule (accept reversed numerator terms) ***(M1)***

eg $\frac{vu'-uv'}{v^2}$, $\frac{uv'-vu'}{v^2}$

evidence of correct differentiation inside the quotient rule ***(A1)(A1)***

eg $f'(x) = \frac{(1+50\mathrm{e}^{-0.2x})(0)-100(50\mathrm{e}^{-0.2x}\times-0.2)}{(1+50\mathrm{e}^{-0.2x})^2}$, $\frac{100(-10)\mathrm{e}^{-0.2x}-0}{(1+50\mathrm{e}^{-0.2x})^2}$

any correct expression for derivative (0 may not be explicitly seen) ***(A1)***

eg $\frac{-100(50\mathrm{e}^{-0.2x}\times-0.2)}{(1+50\mathrm{e}^{-0.2x})^2}$

correct working clearly leading to the required answer ***A1***

eg $f'(x) = \frac{0-100(-10)\mathrm{e}^{-0.2x}}{(1+50\mathrm{e}^{-0.2x})^2}$, $\frac{-100(-10)\mathrm{e}^{-0.2x}}{(1+50\mathrm{e}^{-0.2x})^2}$

$f'(x) = \frac{1000\mathrm{e}^{-0.2x}}{(1+50\mathrm{e}^{-0.2x})^2}$ ***AG N0***

[5 marks]

9e. Find the maximum rate of change of f . *[4 marks]*

Markscheme

METHOD 1

sketch of $f'(x)$ ***(A1)***

eg

recognizing maximum on $f'(x)$ ***(M1)***

eg dot on max of sketch

finding maximum on graph of $f'(x)$ ***A1***

eg $(19.6, 5)$, $x = 19.560\ldots$

maximum rate of increase is 5 ***A1 N2***

METHOD 2

recognizing $f''(x) = 0$ ***(M1)***

finding any correct expression for $f''(x) = 0$ ***(A1)***

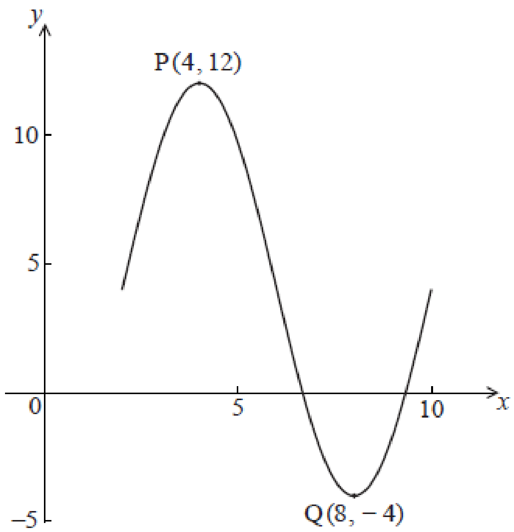
eg $\frac{(1+50e^{-0.2x})^2(-200e^{-0.2x})-(1000e^{-0.2x})(2(1+50e^{-0.2x})(-10e^{-0.2x}))}{(1+50e^{-0.2x})^4}$

finding $x = 19.560\ldots$ ***A1***

maximum rate of increase is 5 ***A1 N2***

[4 marks]

The following diagram shows the graph of $f(x) = a\sin(b(x-c)) + d$, for $2 \le x \le 10$.



There is a maximum point at P(4, 12) and a minimum point at Q(8, −4) .

10a. Use the graph to write down the value of *[3 marks]*

(i) a ;

(ii) c ;

(iii) d .

Markscheme

(i) $a = 8$ ***A1 N1***

(ii) $c = 2$ ***A1 N1***

(iii) $d = 4$ ***A1 N1***

[3 marks]

10b. Show that $b = \frac{\pi}{4}$. *[2 marks]*

Markscheme

METHOD 1

recognizing that period $= 8$ ***(A1)***

correct working ***A1***

e.g. $8 = \frac{2\pi}{b}$, $b = \frac{2\pi}{8}$

$b = \frac{\pi}{4}$ ***AG N0***

METHOD 2

attempt to substitute ***M1***

e.g. $12 = 8\sin(b(4-2)) + 4$

correct working ***A1***

e.g. $\sin 2b = 1$

$b = \frac{\pi}{4}$ ***AG N0***

[2 marks]

10c. Find $f'(x)$. *[3 marks]*

Markscheme

evidence of attempt to differentiate or choosing chain rule ***(M1)***

e.g. $\cos\frac{\pi}{4}(x-2)$, $\frac{\pi}{4} \times 8$

$f'(x) = 2\pi\cos\left(\frac{\pi}{4}(x-2)\right)$ (accept $2\pi\cos\frac{\pi}{4}(x-2)$) ***A2 N3***

[3 marks]

10d. At a point R, the gradient is -2π . Find the *x*-coordinate of R. *[6 marks]*

Markscheme

recognizing that gradient is $f'(x)$ ***(M1)***

e.g. $f'(x) = m$

correct equation ***A1***

e.g. $-2\pi = 2\pi\cos\left(\frac{\pi}{4}(x-2)\right)$, $-1 = \cos\left(\frac{\pi}{4}(x-2)\right)$

correct working ***(A1)***

e.g. $\cos^{-1}(-1) = \frac{\pi}{4}(x-2)$

using $\cos^{-1}(-1) = \pi$ (seen anywhere) ***(A1)***

e.g. $\pi = \frac{\pi}{4}(x-2)$

simplifying ***(A1)***

e.g. $4 = (x-2)$

$x = 6$ ***A1 N4***

[6 marks]

Let $g(x) = \frac{\ln x}{x^2}$, for $x > 0$.

11a. Use the quotient rule to show that $g'(x) = \frac{1-2\ln x}{x^3}$. *[4 marks]*

Markscheme

$\frac{\mathrm{d}}{\mathrm{d}x}\ln x = \frac{1}{x}$, $\frac{\mathrm{d}}{\mathrm{d}x}x^2 = 2x$ (seen anywhere) ***A1A1***

attempt to substitute into the quotient rule (do **not** accept product rule) ***M1***

e.g. $\frac{x^2\left(\frac{1}{x}\right)-2x\ln x}{x^4}$

correct manipulation that clearly leads to result ***A1***

e.g. $\frac{x-2x\ln x}{x^4}$, $\frac{x(1-2\ln x)}{x^4}$, $\frac{x}{x^4}$, $\frac{2x\ln x}{x^4}$

$g'(x) = \frac{1-2\ln x}{x^3}$ ***AG N0***

[4 marks]

11b. The graph of *g* has a maximum point at A. Find the *x*-coordinate of A. *[3 marks]*

Markscheme

evidence of setting the derivative equal to zero ***(M1)***

e.g. $g'(x) = 0$, $1 - 2\ln x = 0$

$\ln x = \frac{1}{2}$ ***A1***

$x = \mathrm{e}^{\frac{1}{2}}$ ***A1 N2***

[3 marks]

12. Let $h(x) = \frac{6x}{\cos x}$. Find $h'(0)$. *[6 marks]*

Markscheme

METHOD 1 (quotient)

derivative of numerator is 6 ***(A1)***

derivative of denominator is $-\sin x$ ***(A1)***

attempt to substitute into quotient rule ***(M1)***

correct substitution ***A1***

e.g. $\frac{(\cos x)(6)-(6x)(-\sin x)}{(\cos x)^2}$

substituting $x=0$ ***(A1)***

e.g. $\frac{(\cos 0)(6)-(6\times 0)(-\sin 0)}{(\cos 0)^2}$

$h'(0)=6$ ***A1 N2***

METHOD 2 (product)

$h(x)=6x\times(\cos x)^{-1}$

derivative of $6x$ is 6 ***(A1)***

derivative of $(\cos x)^{-1}$ is $(-(\cos x)^{-2}(-\sin x))$ ***(A1)***

attempt to substitute into product rule **(M1)**

correct substitution ***A1***

e.g. $(6x)(-(\cos x)^{-2}(-\sin x))+(6)(\cos x)^{-1}$

substituting $x=0$ ***(A1)***

e.g. $(6\times 0)(-(\cos 0)^{-2}(-\sin 0))+(6)(\cos 0)^{-1}$

$h'(0)=6$ ***A1 N2***

[6 marks]

Consider $f(x)=x^2+\frac{p}{x}$, $x\neq 0$, where p is a constant.

13a. Find $f'(x)$. *[2 marks]*

Markscheme

$f'(x) = 2x - \frac{p}{x^2}$ ***A1A1 N2***

Note: Award ***A1*** for $2x$, ***A1*** for $-\frac{p}{x^2}$.

[2 marks]

13b. There is a minimum value of $f(x)$ when $x = -2$. Find the value of p . *[4 marks]*

Markscheme

evidence of equating derivative to 0 (seen anywhere) ***(M1)***

evidence of finding $f'(-2)$ (seen anywhere) ***(M1)***

correct equation ***A1***

e.g. $-4 - \frac{p}{4} = 0$, $-16 - p = 0$

$p = -16$ ***A1 N3***

[4 marks]

Let $f(x) = \cos 2x$ and $g(x) = \ln(3x - 5)$.

14a. Find $f'(x)$. *[2 marks]*

Markscheme

(a) $f'(x) = -\sin 2x \times 2 (= -2\sin 2x)$ ***A1A1 N2***

Note: Award ***A1*** for 2, ***A1*** for $-\sin 2x$.

[2 marks]

14b. Find $g'(x)$. *[2 marks]*

Markscheme

$g'(x) = 3 \times \frac{1}{3x-5} \left(= \frac{3}{3x-5}\right)$ ***A1A1 N2***

Note: Award ***A1*** for 3, ***A1*** for $\frac{1}{3x-5}$.

[2 marks]

14c. Let $h(x) = f(x) \times g(x)$. Find $h'(x)$. *[2 marks]*

Markscheme

evidence of using product rule ***(M1)***

$h'(x) = (\cos 2x)\left(\frac{3}{3x-5}\right) + \ln(3x-5)(-2\sin 2x)$ ***A1 N2***

[2 marks]

15. Consider the curve with equation $f(x) = px^2 + qx$, where *p* and *q* are constants. The point $\mathrm{A}(1{,}3)$ lies on the curve. The tangent to the curve at A has gradient 8. Find the value of *p* and of *q* . *[7 marks]*

Markscheme

substituting $x = 1$, $y = 3$ into $f(x)$ ***(M1)***

$3 = p + q$ ***A1***

finding derivative ***(M1)***

$f'(x) = 2px + q$ ***A1***

correct substitution, $2p + q = 8$ ***A1***

$p = 5$, $q = -2$ ***A1A1 N2N2***

[7 marks]

16. Let $f(x) = \mathrm{e}^x \cos x$. Find the gradient of the normal to the curve of *f* at $x = \pi$. *[6 marks]*

Markscheme

evidence of choosing the product rule ***(M1)***

$f'(x) = \mathrm{e}^x \times (-\sin x) + \cos x \times \mathrm{e}^x \ (= \mathrm{e}^x \cos x - \mathrm{e}^x \sin x)$ ***A1A1***

substituting π ***(M1)***

e.g. $f'(\pi) = \mathrm{e}^\pi \cos \pi - \mathrm{e}^\pi \sin \pi$, $\mathrm{e}^\pi(-1-0)$, $-\mathrm{e}^\pi$

taking negative reciprocal ***(M1)***

e.g. $-\frac{1}{f'(\pi)}$

gradient is $\frac{1}{\mathrm{e}^\pi}$ ***A1 N3***

[6 marks]

Let $f(x) = \mathrm{e}^{-3x}$ and $g(x) = \sin\left(x - \frac{\pi}{3}\right)$.

17a. Write down *[2 marks]*

(i) $f'(x)$;

(ii) $g'(x)$.

Markscheme

(i) $-3\mathrm{e}^{-3x}$ ***A1 N1***

(ii) $\cos\left(x - \frac{\pi}{3}\right)$ ***A1 N1***

[4 marks]

17b. Let $h(x) = \mathrm{e}^{-3x} \sin\left(x - \frac{\pi}{3}\right)$. Find the exact value of $h'\left(\frac{\pi}{3}\right)$. *[4 marks]*

Markscheme

evidence of choosing product rule ***(M1)***

e.g. $uv' + vu'$

correct expression ***A1***

e.g. $-3\mathrm{e}^{-3x}\sin\left(x - \frac{\pi}{3}\right) + \mathrm{e}^{-3x}\cos\left(x - \frac{\pi}{3}\right)$

complete correct substitution of $x = \frac{\pi}{3}$ ***(A1)***

e.g. $-3\mathrm{e}^{-3\frac{\pi}{3}}\sin\left(\frac{\pi}{3} - \frac{\pi}{3}\right) + \mathrm{e}^{-3\frac{\pi}{3}}\cos\left(\frac{\pi}{3} - \frac{\pi}{3}\right)$

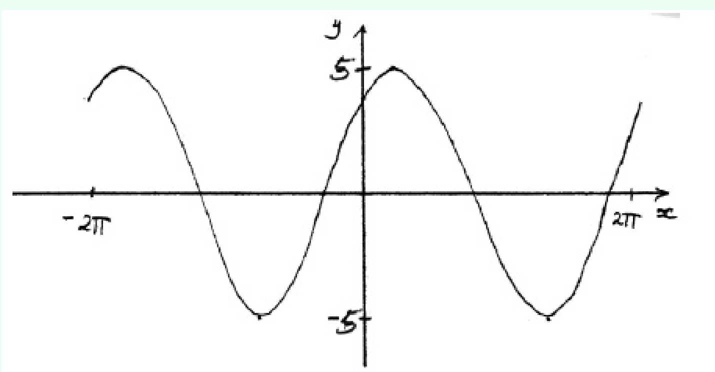
$h'\left(\frac{\pi}{3}\right) = \mathrm{e}^{-\pi}$ ***A1 N3***

[4 marks]

Let $f(x) = 3\sin x + 4\cos x$, for $-2\pi \le x \le 2\pi$.

18a. Sketch the graph of *f* . *[3 marks]*

Markscheme



A1A1A1 N3

Note: Award ***A1*** for approximately sinusoidal shape, ***A1*** for end points approximately correct $(-2\pi, 4)$ $(2\pi, 4)$, ***A1*** for approximately correct position of graph, (*y*-intercept $(0, 4)$, maximum to right of *y*-axis).

[3 marks]

18b. Write down *[3 marks]*

(i) the amplitude;

(ii) the period;

(iii) the x-intercept that lies between $-\frac{\pi}{2}$ and 0.

Markscheme

(i) 5 ***A1 N1***

(ii) 2π (6.28) ***A1 N1***

(iii) -0.927 ***A1 N1***

[3 marks]

18c. Hence write $f(x)$ in the form $p\sin(qx+r)$. *[3 marks]*

Markscheme

$f(x) = 5\sin(x+0.927)$ (accept $p=5$, $q=1$, $r=0.927$) ***A1A1A1 N3***

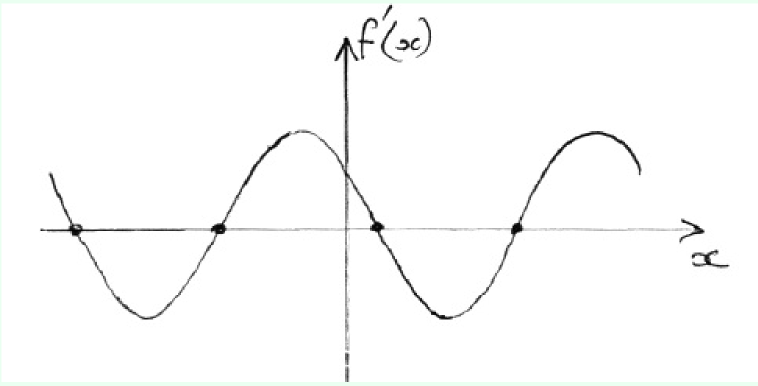
[3 marks]

18d. Write down one value of x such that $f'(x)=0$. *[2 marks]*

Markscheme

evidence of correct approach ***(M1)***

e.g. max/min, sketch of $f'(x)$ indicating roots



one 3 s.f. value which rounds to one of -5.6, -2.5, 0.64, 3.8 ***A1 N2***

[2 marks]

18e. Write down the two values of *k* for which the equation $f(x) = k$ has exactly two solutions. *[2 marks]*

Markscheme

$k = -5$, $k = 5$ ***A1A1 N2***

[2 marks]

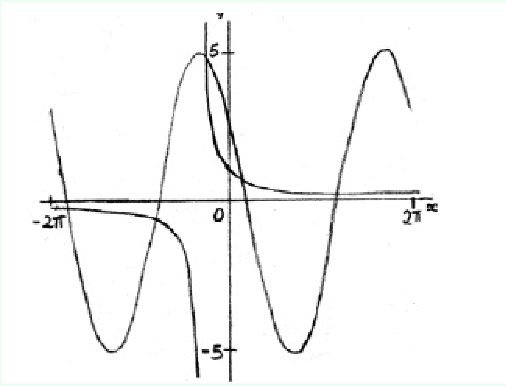
18f. Let $g(x) = \ln(x + 1)$, for $0 \le x \le \pi$. There is a value of *x*, between 0 and 1, for which the gradient of *f* is equal to the gradient of *g*. Find this value of *x*. *[5 marks]*

Markscheme

METHOD 1

graphical approach (but must involve derivative functions) ***M1***

e.g.



each curve ***A1A1***

$x = 0.511$ ***A2 N2***

METHOD 2

$g'(x) = \frac{1}{x+1}$ ***A1***

$f'(x) = 3\cos x - 4\sin x \ (5\cos(x + 0.927))$ ***A1***

evidence of attempt to solve $g'(x) = f'(x)$ ***M1***

$x = 0.511$ ***A2 N2***

[5 marks]

Consider the curve $y = \ln(3x - 1)$. Let P be the point on the curve where $x = 2$.

19a. Write down the gradient of the curve at P. *[2 marks]*

Markscheme

gradient is 0.6 ***A2 N2***

[2 marks]

19b. The normal to the curve at P cuts the *x*-axis at R. Find the coordinates of R. *[5 marks]*

Markscheme

at R, $y = 0$ (seen anywhere) ***A1***

at $x = 2$, $y = \ln 5\ (= 1.609\ldots)$ ***(A1)***

gradient of normal $= -1.6666\ldots$ ***(A1)***

evidence of finding correct equation of normal ***A1***

e.g. $y = \ln 5 = -\frac{5}{3}(x - 2)$, $y = -1.67x + c$

$x = 2.97$ (accept 2.96) ***A1***

coordinates of R are (2.97,0) ***N3***

[5 marks]