Intro to Calculus Review *[158 marks]*

The values of the functions f and g and their derivatives for $x = 1$ and $x = 8$ are shown in the following table.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	4	9	–3
8	4	–3	2	5

Let $h(x) = f(x)g(x)$.

1a. Find $h(1)$. *[2 marks]*

1b. Find $h'(8)$. *[3 marks]*

2. Let $f(x) = (x^2 + 3)^7$. Find the term in x^5 in the expansion of the derivative, $f'(x)$. *[7 marks]*

3. Let $f(x) = \frac{\ln(4x)}{x}$ for $0 < x \leq 5$. *[7 marks]*

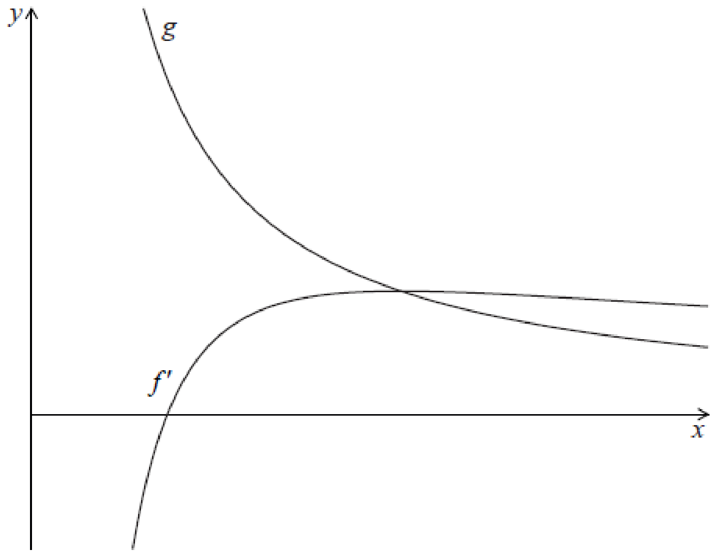
Points $\mathrm{P}(0.25, 0)$ and Q are on the curve of f. The tangent to the curve of f at P is perpendicular to the tangent at Q. Find the coordinates of Q.

Let $f(x) = \frac{(\ln x)^2}{2}$, for $x > 0$.

4a. Show that $f'(x) = \frac{\ln x}{x}$. *[2 marks]*

4b. There is a minimum on the graph of f. Find the x-coordinate of this minimum. *[3 marks]*

Let $g(x) = \frac{1}{x}$. The following diagram shows parts of the graphs of f' and g.



The graph of f' has an x-intercept at $x = p$.

. Write down the value of p. *[2 marks]*

The graph of g intersects the graph of f' when $x = q$. *[3 marks]*

Find the value of q.

5. Let $f(x) = \mathrm{e}^{2x}$. The line L is the tangent to the curve of f at $(1, \mathrm{e}^2)$. *[6 marks]*

Find the equation of L in the form $y = ax + b$.

Let $f(x) = \sqrt[3]{x^4} - \frac{1}{2}$.

6. Find $f'(x)$. *[2 marks]*

Consider $f(x) = x^2 \sin x$.

7a. Find $f'(x)$. *[4 marks]*

7b. Find the gradient of the curve of f at $x = \frac{\pi}{2}$. *[3 marks]*

Let $f(x) = \sin x + \frac{1}{2}x^2 - 2x$, for $0 \leq x \leq \pi$.

8a. Find $f'(x)$. *[3 marks]*

Let g be a quadratic function such that $g(0) = 5$. The line $x = 2$ is the axis of symmetry of the graph of g .

8b. Find $g(4)$. *[3 marks]*

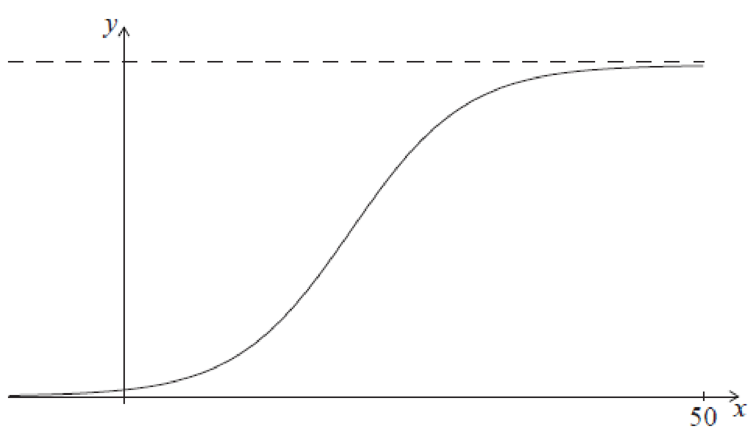
The function g can be expressed in the form $g(x) = a(x-h)^2 + 3$.

8c. (i) Write down the value of h . *[4 marks]*

(ii) Find the value of a .

8d. Find the value of x for which the tangent to the graph of f is parallel to the tangent to the graph of g . *[6 marks]*

Let $f(x) = \frac{100}{(1+50\mathrm{e}^{-0.2x})}$. Part of the graph of f is shown below.



9a. Write down $f(0)$. *[1 mark]*

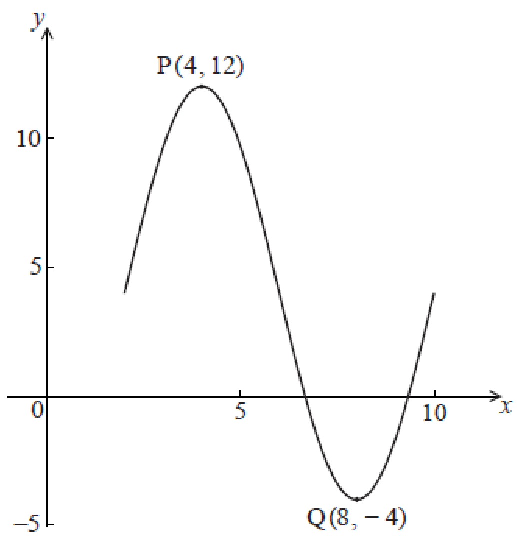
9b. Solve $f(x) = 95$. *[2 marks]*

9c. Find the range of f . *[3 marks]*

9d. Show that $f'(x) = \frac{1000e^{-0.2x}}{(1+50e^{-0.2x})^2}$. *[5 marks]*

9e. Find the maximum rate of change of f . *[4 marks]*

The following diagram shows the graph of $f(x) = a\sin(b(x-c)) + d$, for $2 \le x \le 10$.



There is a maximum point at P(4, 12) and a minimum point at Q(8, −4) .

10a. Use the graph to write down the value of *[3 marks]*

(i) *a* ;

(ii) *c* ;

(iii) *d* .

10b. Show that $b = \frac{\pi}{4}$. *[2 marks]*

10c. Find $f'(x)$. *[3 marks]*

10d. At a point R, the gradient is -2π . Find the *x*-coordinate of R. *[6 marks]*

Let $g(x) = \frac{\ln x}{x^2}$, for $x > 0$.

11a. Use the quotient rule to show that $g'(x) = \frac{1-2\ln x}{x^3}$. *[4 marks]*

11b. The graph of g has a maximum point at A. Find the x-coordinate of A. *[3 marks]*

12. Let $h(x) = \frac{6x}{\cos x}$. Find $h'(0)$. *[6 marks]*

Consider $f(x) = x^2 + \frac{p}{x}$, $x \neq 0$, where p is a constant.

13a. Find $f'(x)$. *[2 marks]*

13b. There is a minimum value of $f(x)$ when $x = -2$. Find the value of p . *[4 marks]*

Let $f(x) = \cos 2x$ and $g(x) = \ln(3x - 5)$.

14a. Find $f'(x)$. *[2 marks]*

14b. Find $g'(x)$. *[2 marks]*

14c. Let $h(x) = f(x) \times g(x)$. Find $h'(x)$. *[2 marks]*

15. Consider the curve with equation $f(x) = px^2 + qx$, where p and q are constants. The point $\mathrm{A}(1,3)$ lies on the curve. The tangent to the curve at A has gradient 8. Find the value of p and of q . *[7 marks]*

16. Let $f(x) = \mathrm{e}^x \cos x$. Find the gradient of the normal to the curve of f at $x = \pi$. *[6 marks]*

Let $f(x) = \mathrm{e}^{-3x}$ and $g(x) = \sin\left(x - \frac{\pi}{3}\right)$.

17a. Write down *[2 marks]*

(i) $f'(x)$;

(ii) $g'(x)$.

17b. Let $h(x) = \mathrm{e}^{-3x} \sin\left(x - \frac{\pi}{3}\right)$. Find the exact value of $h'\left(\frac{\pi}{3}\right)$. *[4 marks]*

Let $f(x) = 3\sin x + 4\cos x$, for $-2\pi \leq x \leq 2\pi$.

18a. Sketch the graph of f . *[3 marks]*

18b. Write down *[3 marks]*

(i) the amplitude;

(ii) the period;

(iii) the x-intercept that lies between $-\frac{\pi}{2}$ and 0.

18c. Hence write $f(x)$ in the form $p\sin(qx + r)$. *[3 marks]*

18d. Write down one value of x such that $f'(x) = 0$. *[2 marks]*

18e. Write down the two values of k for which the equation $f(x) = k$ has exactly two solutions. *[2 marks]*

18f. Let $g(x) = \ln(x + 1)$, for $0 \leq x \leq \pi$. There is a value of x, between 0 and 1, for which the gradient of f is equal to the gradient of g. Find this value of x. *[5 marks]*

Consider the curve $y = \ln(3x - 1)$. Let P be the point on the curve where $x = 2$.

19a. Write down the gradient of the curve at P. *[2 marks]*

19b. The normal to the curve at P cuts the x-axis at R. Find the coordinates of R. *[5 marks]*